

Find the derivatives of the following

1. $y = 9$

$$\frac{d}{dx}(9) = 0$$

Derivative of a constant function is 0

2. $y = \ln(3x)$

This can be rewritten as $(\ln(3x))' = (\ln(x) + \ln(3))'$

$$(\ln(x) + \ln(3))' = (\ln(3))' + (\ln(x))'$$

Derivative of a natural logarithm is $(\ln(x))' = \frac{1}{x}$

Derivative of a constant = 0

Hence

$$(\ln(3))' + (\ln(x))' = (0) + \frac{1}{x}$$

$$\text{Therefore, } (\ln(3x))' = \frac{1}{x}$$

3. $y = ex$

$$\frac{d}{dx}(ex)$$

$$\text{using constant rule, } C \times \frac{d}{dx}(x) = e \times \frac{d}{dx}(x)$$

Also, using the power rule, $(\frac{d}{dx}(x^n) = n * x^{-1+n})$. With $n = 1$, we have $\frac{dy}{dx}(x) = 1$

$$= e \frac{dy}{dx}(x) = e(1)$$

$$\frac{d}{dx}(ex) = e$$

4. $y = \cos(\ln(2x))$

The function is composed of $f(g(x))$ and $f(u) = \cos(u)$

And $u = g(x) = \ln(x) + \ln(2)$

$$\text{Using chain rule, } (f(g(x)))' = \frac{d}{du}(f(u)) \cdot (g(x))'$$

$$= (\cos(\ln(x) + \ln(2)))' = \frac{d}{du}(\cos(u)) \frac{d}{du}(\ln(x) + \ln(2))$$

The derivative of cosine is $\frac{d}{du}(\cos(u)) = -\sin u$

$$\frac{d}{du}(\cos(u)) \frac{d}{dx}(\ln(x) + \ln(2)) = (-\sin(u)) \frac{d}{dx}(\ln(x) + \ln(2)) = -\sin$$

$$(u) \frac{d}{dx}(\ln(2x))$$

$$= -\sin(u) \frac{d}{dx}(\ln(2x)) = -\sin((\ln(x) + \ln(2))) \frac{d}{dx}(\ln(2x))$$

$$= -\sin(\ln(x) + \ln(2)) \frac{d}{dx}(\ln(2x)) = -\sin(\ln(x) + \ln(2)) \frac{d}{dx}(\ln(2x))$$

$$= -\sin(\ln(2x)) \frac{d}{dx}(\ln(2x))$$

The derivative of a natural logarithm is $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$

$$-\sin(\ln(2x)) \frac{d}{dx}(\ln(2x)) = -\sin(\ln(2x)) \frac{d}{du}(\ln(u)) \frac{d}{dx}(2x)$$

The derivative of a natural logarithm is $\frac{d}{du}(\ln(u)) = \frac{1}{u}$

$$-\sin(\ln(2x)) \frac{d}{du}(\ln(u)) \frac{d}{dx}(2x) = -\sin(\ln(2x)) \frac{1}{u} \frac{d}{dx}(2x)$$

$$-\frac{\frac{d}{dx}(2x)}{u} \sin(\ln(2x)) = -\frac{\sin(\ln(2x))}{(2x)} \frac{d}{dx}(2x)$$

Using the constant multiple rules, we have

$$-\frac{\sin(\ln(2x))}{2x} \frac{d}{dx}(2x) = -\frac{\sin(\ln(2x))}{2x} (2 \frac{d}{dx}(x))$$

$$-\frac{\frac{d}{dx}(x)}{x} \sin(\ln(2x)) = -\frac{1}{x} \sin(\ln(2x))$$

$$\frac{d}{dx}(\cos(\ln(2x))) = -\frac{1}{x} \sin(\ln(2x))$$

5. $y = e \cot x$

Using constant multiple rule

$$= \frac{d}{dx}(e \cot(x)) = e \frac{d}{dx}(\cot(x))$$

The derivative of cotangent is $\frac{d}{dx}(\cot(x)) = -\csc^2(x)$

$$\text{Therefore, } \frac{d}{dx} e \cot(x) = -\csc^2(x)$$

6. $y = \ln(4x + x)$

$$\ln(4x + x) = \ln(5x)$$

$$\frac{d}{dx}(\ln(5x)) = \frac{d}{dx}(\ln(x) + \ln(5))$$

$$= \frac{d}{dx}(\ln(x) + \ln(5)) = \frac{d}{dx}((\ln(5) + (\ln(x)))$$

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\ln(x)) + \frac{d}{dx}(\ln(5)) = \frac{1}{x} + \frac{d}{dx}(\ln(5))$$

$$\text{Thus } \frac{d}{dx}(\ln(x)) + \frac{d}{dx}(\ln(5)) = \frac{1}{x}$$

$$\text{Therefore, } \frac{d}{dx}(\ln(5x)) = \frac{1}{x}$$

7. $\cos(x + y) = \ln(x)$

$$\frac{d}{dx}(-\ln(x) + \cos(x + y)) = (\frac{d}{dx}(\ln(x))) + \frac{d}{dx}(x + y)$$

$$\frac{d}{dx}(\cos(x + y)) - \frac{d}{dx}(\ln(x)) = \frac{d}{du}(\cos(u)) \frac{d}{dx}(x + y) - \frac{d}{dx}(\ln(x))$$

$$\frac{d}{du}(\cos(u)) \frac{d}{dx}(x + y) - \frac{d}{dx}(\ln(x)) = (-\sin(u)) \frac{d}{dx}(x + y) - \frac{d}{dx}(\ln(x))$$

The derivative of a sum difference is the sum difference of the derivatives:

$$\begin{aligned}
& -\sin(x+y) \frac{d}{dx}(x+y) - \frac{d}{dx}(\ln(x)) \\
& = -\sin(x+y) \left(\frac{d}{dx}(x) + \frac{d}{dx}(y) \right) - \frac{d}{dx}(\ln(x))
\end{aligned}$$

The derivative of a constant is zero, and derivative of a natural logarithm is $\frac{dy}{dx}(\ln(x)) = \frac{1}{x}$

$$-\sin(x+y) \frac{d}{dx}(x) - \frac{d}{dx}(\ln(x)) = -\sin(x+y) \frac{d}{dx}(x) - \frac{1}{x}$$

$$-\sin(x+y) \frac{d}{dx}(x+y)(x) - \frac{1}{x} = -\sin(x+y)(1) - \frac{1}{x}$$

$$\text{Thus, } \frac{d}{dx}(-\ln(x) + \cos(x+y)) = -\sin(x+y) - \frac{1}{x}$$

$$\text{Therefore, } \frac{d}{dx}(-\ln(x) + \cos(x+y)) = -\sin(x+y) - \frac{1}{x}$$

$$8. \quad y = \log(\sin x)$$

The function is composition of $f(g(x))$ of two functions $f(u) = \ln(u)$ and $u = g(x) = \sin(x)$

Using chain rule, we have $\frac{d}{dx}(\ln(\sin(x))) = \frac{d}{du}(\ln(u)) \frac{d}{dx}(\sin(x)) = \frac{1}{u} \frac{d}{dx}(\sin(x))$

$$= \frac{\frac{d}{dx}(\sin(x))}{u} = \frac{\frac{d}{dx}(\sin(x))}{\sin(x)}$$

The derivative of a sine is $\frac{d}{dx}(\sin(x)) = \cos(x)$

$$\frac{\frac{d}{dx}(\sin(x))}{\sin(x)} = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$

$$\text{Therefore, } \frac{d}{dx}(\ln(\sin(x))) = \frac{1}{\tan(x)}$$